Maximal Multipolarized Cliques Search in Signed Networks

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ABSTRACT

The increasing of group polarization on social media seriously impacts on the health of public discourse and information dissemination. At present, detecting polarized structures in signed networks is well-motivated for studying the group polarization on social media. However, most studies restricted the number of polarized structures to only two, while neglecting the real-world scenario where signed networks consist of multiple polarized structures, that is an unreasonable assumption. To conquer the limitations of the existing work, in this paper, we present a novel cohesive subgraph model based on structural clusterable theory, named maximal multipolarized clique (MMC), which can be partitioned into k polarized subcliques such that the edges in subcliques are positive and the edges between subcliques are negative. This paper formulates the problem of Maximal Multipolarized Cliques Search (MMCS) in signed networks which is proved to be NP-hard. To address this problem, we first devise powerful pruning rules to reduce the signed network significantly and further develop an efficient algorithm to search all maximal multipolarized cliques in the reduced signed network. The experimental results on real-world signed networks demonstrate the efficiency and effectiveness of our algorithm.

CCS CONCEPTS

• **Human-centered computing** → *Social network analysis.*

KEYWORDS

Group Polarization; Signed Networks; Maximal Multipolarized Clique

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1 INTRODUCTION

Nowadays, group polarization around controversial social media issues has become prevalent and is increasingly recognized as a deeply concerning problem. There is no doubt that social media algorithms accurately recommend and continuously display filtered information to meet user preferences, thereby helping the platform attract a multitude of users. However, users prefer to see opinions or logic similar to theirs on social media, while ignoring any information that may contradict their preexisting ideas. This quickly causes polarization and then leads to the formation of multiple polarized “echo chambers". Excessive polarized “echo chambers" help users strengthen their beliefs while alienating those who hold different views, which provides a negative user experience and seriously impacts the health of public discourse and information circulation. Therefore, mining polarized structures in social networks is well-motivated for paving the way to further study group polarization [9].

Nevertheless, how to represent the attitude or sentiment of interactions between individuals? It should be acknowledged that a signed network can powerfully encode many real-world relations between two entities with positive and negative links, such as pros-cons opinions in opinion networks and friend-fore relationships between users in social networks [5]. A great deal of recent research has focused on discovering polarized communities in signed networks [1, 6, 9]. For example, the study [1] has formulated and researched the problem of discovering two polarized communities in signed networks based on graph spectral methods. However, these methods aim to find two communities in a global signed network. In the literature, considerable approaches [3, 5, 7, 8] have been proposed for cohesive subgraph detection in signed networks. For example, the balanced clique model of signed networks is proposed in [3]. A balance clique is defined as a maximal clique that can be partitioned into two subcliques in which the edges in subclique are positive and the edges between subcliques are negative.

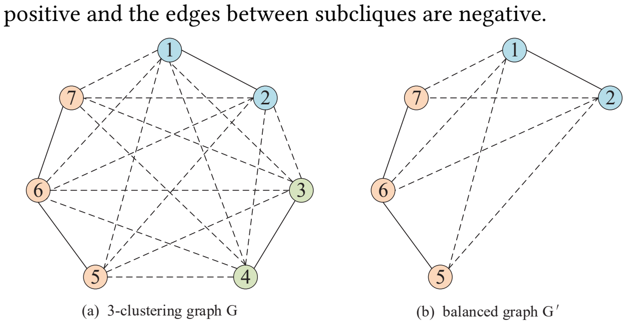

Figure 1: An example of k-clustering graph and balanced graph. Solid edges are positive, dashed edges are negative.

However, in the existing works[1, 3, 9], restricting the number of polarized structures to only two is an unreasonable assumption to make while representing multiple polarized structures in real-world signed networks, which is actually influenced by *structural balance* theory [2] in signed network analysis. In fact, [4] has already extended structural balance theory and proposed the other

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fundamental theory, named *structural clusterable* theory, including a new concept of k-clustering graph to relax the strict constraint. Specifically, compared with a balanced graph divided into two mutually exclusive subgraphs, a k-clustering graph can be partitioned into k opposing clustering such that the edges in the same clustering are positive and the edges between clustering are negative. For example, Figure 1(a) presents a 3-clustering graph which can be split into three clustering, i.e., $\{1, 2\}$, $\{3, 4\}$ and $\{5, 6, 7\}$. If we restrict k to 2, we can obtain a balanced graph $\{1, 2, 5, 6, 7\}$ consisting only two polarized clustering $\{1, 2\}$ and $\{5, 6, 7\}$ in Figure 1(b). Obviously, the k-clustering graph model is more flexible than balanced graph in signed networks.

Drawing inspiration from the above-mentioned work, we propose the maximal multipolarized clique for signed networks in this paper that considers the structural clusterable theory and study the MMCS problem which searches all the maximal multipolarized cliques in a given signed network. An MMC satisfies three properties: (i) it is a clique in which every pair of nodes have an edge. (ii) it can be partitioned into k subcliques, and every subclique's size is no less than a. (iii) it is maximal, i.e., none of its supergraphs meets the conditions (i) and (ii). The MMC model sets the arbitrary number of polarized groups that conforms to structural clusterable of real-world signed networks, and also compacts the clique property which considers the homogeneity of social networks. From an application perspective, compared with the existing models, the proposed MMC model with an arbitrary number of polarized structures is more suitable for real-world signed networks. It can also be used in many applications, such as opinion leader detection, trust community mining, protein complex discovery, etc [5].

To the best of our knowledge, this is the first work to propose the MMC model considering structural clusterable theory and further study the MMCS problem in signed networks. We also prove that the MMCS problem in signed networks is NP-hard. To solve this problem, we first devise powerful signed network reduction techniques to significantly prune the large-scale signed networks. An efficient enumeration algorithm is then developed with tailored pruning rules to enumerate all MMCs in signed network. Finally, we conduct extensive experiments on real-world signed networks to demonstrate the efficiency and effectiveness of this new algorithm.

2 PROBLEM STATEMENT

Let $G = (V, E^+, E^-)$ be an undirected signed graph, where V is the set of nodes, E^+ and E^- are the sets of positive and negative edges in G, respectively. $n = |V|$ and $m = |E^+|+|E^-|$ are the number of nodes and edges. For each edge $e \in E$, it is associated with a label either "+" or "-". For each node $u \in V$, let $N_u = \{v|(u, v) \in E, u, v \in V\}$ be the set of neighbor nodes of u, $N_u^+ = \{v|(u, v) \in E^+, u, v \in V\}$ be the set of positive neighbors, and $N_u^- = \{v|(u, v) \in E^-, u, v \in V\}$ be the set of negative neighbors. Let $d_u = |N_u|$, $d_u^+ = |N_u^+|$, $d_u^- = |N_u^-|$ be the degree, the positive degree, and the negative degree of u, respectively.

Definition 2.1. **(Maximal Multipolarized Clique)** Given a signed network $G = \{V, E^+, E^-\}$ and two integers k and α, an MMC C is a maximal subgraph of G that satisfies the following constraints:

• **Clique Constraint**: C is complete, i.e, $\forall u, v \in C \Rightarrow (u, v) \in E^+ \cup E^-$.

• **Polarized Constraint**: C can be partitioned into k polarized subcliques $P_1, P_2...P_k$, s.t. $\forall e \in \{(u, v)|u, v \in P_i, i \in [1, k]\} \Leftrightarrow e \in E^+$, and $\forall e \in \{(u, v)|u \in P_i, v \in P_j, i, j \in [1, k], i \neq j\} \Leftrightarrow e \in E^-$. The size of each polarized subclique is no less than α, s.t. $|P_i| \geq \alpha$.

• **Maximal Constraint**: there is no multipolarized clique C' in G containing C.

Intuitively, the clique constraint ensures the subgraph is densely connected. The polarized constraint guarantees the subgraph meets structural clusterable theory and considers the practical application requires a fixed threshold of size.

Example 2.2. Considering the 3-clusterable graph G in Figure 1(a), assume $k = 3$ and $\alpha = 2$, there are two MMCs in G, i.e., $C_1 = \{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$ and $C_2 = \{\{1, 2\}, \{3, 4\}, \{6, 7\}\}$. Taking C_1 as an example, it is a clique since any two nodes in C_1 have an edge. C_1 can be be split into three polarized subcliques, i.e., $\{1, 2\}$, $\{3, 4\}$ and $\{5, 6\}$, and the size of each subclique is no less than 2. It is maximal because if node 7 is added into it, neither the clique constraint nor polarized constraint will not be satisfied.

Problem Statement. Given a signed network $G = (V, E^+, E^-)$ and two positive integers k and α, the goal of MMCS problem is to enumerate all MMCs in G.

Problem Hardness. The MMCS problem is NP-hard. Suppose that $k = 1$, an MMC is degraded to a traditional maximal clique. This is because three constraints in Definition 2.1 always hold when $k = 1$. Therefore, searching all MMCs in G is equivalent to enumerating all traditional maximal cliques if $k = 1$. Thus, the classic maximal clique enumeration problem is a special case of our MMCS problem. Since the traditional maximal clique search problem is NP-hard, MMCS is also NP-hard.

3 APPROACH

To address MMCS problem, several powerful pruning techniques for social networks and an efficient identification algorithm are elaborated below.

3.1 Signed Graph Reduction

In this section, we propose several effective rules to prune the unpromising nodes that are definitely not contained in any MMC.

In [3], the (l, r)-signed core is defined as the maximal subgraph of G such that every node in this subgraph has a positive degree no less than l and a negative degree no less than r, s.t. $\forall u \in G \rightarrow d_u^+ \geq l \wedge d_u^- \geq r$. Next, we show that all MMCs are contained in the specific (l, r)-signed core of G.

Lemma 3.1. *Given a signed network G and two integers k and α, any MMC $C = \{P_1, P_2...P_k\}$ is contained in a connected component of the $(\alpha - 1, (k - 1)\alpha)$-signed core.*

Proof. Based on Definition 2.1, the edges in subcliques P_i are positive and the edges between subcliques P_i and P_j are negative, and $|P_i| \geq \alpha$. Clearly, for each node $u \in C$, it should have the least $\alpha - 1$ positive neighbors in same subclique and the least $(k - 1)\alpha$ negative neighbors in different subcliques, s.t. $\forall u \in C \rightarrow d_u^+ \geq \alpha - 1 \wedge d_u^- \geq (k - 1)\alpha$. Thus, C forms a $(\alpha - 1, (k - 1)\alpha)$-signed core. Since any MMC is connected, it must be contained in a connected component of the $(\alpha - 1, (k - 1)\alpha)$-signed core. □

According to structure clusterable theory [4], a triangle is unclusterable if it contains two positive edges and one negative edge (Figure 2(b)), otherwise, the triangle is clusterable (Figure 2(a)). If all triangles in a graph are clusterable, then the graph is clusterable. Given a node v, we use $\triangle_v^{+++}$, $\triangle_v^{+--}$ and $\triangle_v^{---}$ to denote clusterable triangles that contain v. And the cardinality of above triangles is denoted by $|\triangle_v^{+++}|$, $|\triangle_v^{+--}|$ and $|\triangle_v^{---}|$.

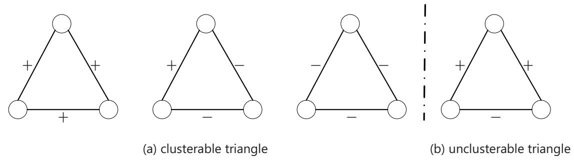


Figure 2: Example of clusterable and unclusterable triangles

Lemma 3.2. *Given a signed network $G = (V, E^+, E^-)$, for an MMC $C = \{P_1, P_2 ... P_k\}$ in G, C is contained in a maximal subgraph G' of G that satisfies the following constraints:*

$$\forall v \in V_{G'} \Rightarrow \begin{cases} |\triangle_v^{+++}| \geq \binom{\alpha-1}{2}, \\ |\triangle_v^{+--}| \geq \binom{\alpha-1}{1}\binom{k-1}{1}\binom{\alpha}{1}, \\ |\triangle_v^{---}| \geq \binom{k-1}{2}\binom{\alpha}{1}\binom{\alpha}{1}. \end{cases}$$

Proof. Based on Definition 2.1, the edges in subcliques are positive and the edges between subcliques are negative. For any node $v \in P_i \in C$, and its two positive neighbors in the same subclique P_i, they form triangles $\triangle_v^{+++}$. Similarly, the triangles $\triangle_v^{---}$ can be formed by v and its two negative neighbors in different subcliques. In addition, the triangles $\triangle_v^{+--}$ can be formed by v and its one positive neighbor in the same subclique and one negative neighbor in different subcliques. Since the size of each P_i is no less than α, $|\triangle_v^{+++}|$, $|\triangle_v^{---}|$ and $|\triangle_v^{+--}|$ are at least $\binom{\alpha-1}{2}$, $\binom{k-1}{2}\binom{\alpha}{1}\binom{\alpha}{1}$ and $\binom{\alpha-1}{1}\binom{k-1}{1}\binom{\alpha}{1}$, respectively. Since the number of triangles meets the requirements of maximal subgraph G' of G, C must be contained in G'. Thus, the above lemma holds. □

Algorithm 1: SGReduction(G, k, α)

1 **Procedure** CoreBased(G, k, α) `// Lemma 3.1`
2 **for** each $v \in V$ **do** compute d_v^+ and d_v^-;
3 **while** $\exists v \in V$, s.t. $d_v^+ < \alpha - 1$ or $d_v^- < (k-1)\alpha$ **do**
4   **for** each $u \in N_v^+$ **do** $d_u^+ \leftarrow d_u^+ - 1$;
5   **for** each $u \in N_v^-$ **do** $d_u^- \leftarrow d_u^- - 1$;
6     $G \leftarrow G \backslash v$;
7 **return** G;
8 **Procedure** TriangleBased(G, k, α) `// Lemma 3.2`
9 **for** each $v \in V$ **do** compute $|\triangle_v^{+++}|$, $|\triangle_v^{+--}|$ and $|\triangle_v^{---}|$;
10 **while** $\exists v \in V$, s.t. $|\triangle_v^{+++}| < \binom{\alpha-1}{2}$ or $|\triangle_v^{+--}| < \binom{\alpha-1}{1}\binom{k-1}{1}\binom{\alpha}{1}$ or $|\triangle_v^{---}| < \binom{k-1}{2}\binom{\alpha}{1}\binom{\alpha}{1}$ **do**
11   **for** each u s.t. $(u, v) \in \triangle_v^{+++}$ **do** $|\triangle_u^{+++}| \leftarrow |\triangle_u^{+++}| - 1$;
12   **for** each u s.t. $(u, v) \in \triangle_v^{+--}$ **do** $|\triangle_u^{+--}| \leftarrow |\triangle_u^{+--}| - 1$;
13   **for** each u s.t. $(u, v) \in \triangle_v^{---}$ **do** $|\triangle_u^{---}| \leftarrow |\triangle_u^{---}| - 1$;
14     $G \leftarrow G \backslash v$;
15 **return** G;

With Lemmas 3.1 and 3.2, a signed graph reduction algorithm (SGReduction) is presented in Algorithm 1. The CoreBased procedure is shown in Line 1-7. We first compute d_v^+ and d_v^- for each vertex of G (line 2). Then the nodes that do not conform to Lemma 3.1 are deleted (line 3) until no such kind of nodes exist (line 3-6). Before v is deleted from G (line 6), we respectively decrease the positive degree of each positive neighbor and the negative degree of each negative neighbor by 1 (line 4-5). At last, the reduced signed network G is returned (line 7). Based on Lemma 3.2, the Triangle-Based procedure is presented in Line 8-15. We first compute $|\triangle_v^{+++}|$, $|\triangle_v^{+--}|$ and $|\triangle_v^{---}|$ for each node (line 9). Similarly, the node that does not satisfy Lemma 3.2 will be deleted until no such kind of nodes exist (line 10). After the corresponding parameters are processed (line 11-13), the node will be safely removed (line 14). At last, the reduced signed network G is returned (line 15).

3.2 The MMCSearch Algorithm

In this section, we present an efficient algorithm with tailored pruning rules to enumerate all MMCs in the reduced signed network.

Theorem 3.3. *Given a signed network $G = (V, E^+, E^-)$, for a multipolarized clique $\mathbb{C} = \{P_1, P_2, P_i ... P_k\}$ in G, if there exists a node v in G such that $\forall u \in P_i \Rightarrow (u, v) \in E^+$ and $\forall w \in \{P_1, P_2 ... P_k\} \backslash P_i \Rightarrow (v, w) \in E^-$, then $\mathbb{C}' = \{P_1, P_2, P_i \cup \{v\} ... P_k\}$ is also a multipolarized clique in G.*

Proof. It can be proved following Definition 2.1 directly. □

According to Theorem 3.3, we can extend the famous Bron-Kerbosch Algorithm, an enumeration algorithm for finding all the maximal cliques in an undirected graph, to search all MMCs in a given signed network. Specifically, if we maintain a temporary multipolarized clique $\mathbb{C} = \{P_1, P_2, P_i ... P_k\}$, let $Q = \{Q_1, Q_2, Q_i ... Q_k\}$ store the candidate vertices where Q_i is the set of vertices that are positive neighbors of all the vertices in $P_i \in \mathbb{C}$ and negative neighbors of all the vertices in $\{P_1, P_2, P_i ... P_k\} \backslash P_i$, we can enlarge $\mathbb{C}$ by adding vertices from Q_i into P_i. Furthermore, if we update Q based on the new $\mathbb{C}' = \{P_1, P_2, P_i \cup \{v\} ... P_k\}$ and repeat the above procedure, an MMC can be obtained when no more vertices can be added into any P_i.

With the above idea, our maximal multipolarized clique search algorithm (MMCSearch) is presented in Algorithm 2. For each vertex v_i in G (line 2), we try to search all MMCs containing v_i (line 1-10). The key unit MMCSearchUnit procedure is shown in Lines 11–20. Note that MMCSearchUnit requires three input parameters $\mathbb{C}$, Q, and R, where $\mathbb{C}$ is initialized to preserve the temporary multipolarized clique (line 2-3), Q is initialized to store the possible candidate vertices (line 4-6) and R is initialized to record the already processed nodes (line 7-9). After that, MMCSearchUnit is invoked (line 10) to search all MMCs. If $\forall Q_i \in Q$ and $\forall R_i \in R$ are empty (line 12) (i.e., current $\mathbb{C}$ is maximal), then it checks whether any $|P_i|$ is no less than α (line 13). If all the constraints are satisfied, it returns the newly detected MMC C (line 14). Otherwise, MMCSearchUnit adds a vertex from Q_i to P_i, updates the corresponding Q_i and R_i (line 15-18), and recursively invokes itself to further extend the multipolarized clique (line 19). When $v \in Q_i$ is processed, v is removed from Q_i and recorded into R_i (line 20).

4 EXPERIMENTS

In this section, we conduct extensive experiments to evaluate the performance of our proposed algorithms on the real-world signed

Algorithm 2: MMCSearch(G, k, α)

for each $v_i \in \{v_1, v_2 ... v_n\} \in V$ **do**
  for each $P_i \in \mathbb{C}$ **do**
    $P_1 \leftarrow \{v_i\}; P_i \leftarrow \emptyset;$
  for each $Q_i \in Q$ **do**
    $Q_1 \leftarrow N_G^+(v) \cap \{v_{i+1}, ..., v_n\};$
    $Q_i \leftarrow N_G^-(v) \cap \{v_{i+1}, ..., v_n\};$
  for $R_i \in R$ **do**
    $R_1 \leftarrow N_G^+(v) \cap \{v_1, ..., v_{i-1}\};$
    $R_i \leftarrow N_G^-(v) \cap \{v_1, ..., v_{i-1}\};$
MMCSearchUnit$(\mathbb{C}, Q, R)$;
Procedure MMCSearchUnit$(\mathbb{C}, Q, R)$
if *all* $Q_i = \emptyset$ *and all* $R_i = \emptyset$ **then**
  if *all* $|P_i| \geq \alpha$ **then**
    return $C = \{P_1, P_2 ... P_k\}$
for each $v \in Q_i$ s.t. $Q_i \neq \emptyset$ **do**
  $P_i \leftarrow P_i \cup \{v\}; Q_i \leftarrow Q_i \cap N_v^+; R_i \leftarrow R_i \cap N_v^+;$
  for each $Q_j \in Q \backslash Q_i$ **do** $Q_j \leftarrow Q_j \cap N_v^-;$
  for each $R_j \in R \backslash R_i$ **do** $R_j \leftarrow R_j \cap N_v^-;$
  MMCSearchUnit$(\mathbb{C}', Q', R')$;
  $Q_i \leftarrow Q_i \backslash \{v\}; R_i \leftarrow R_i \cup \{v\};$

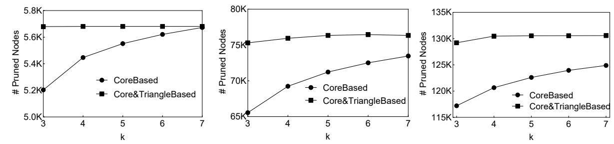
networks. All the experiments are on PC with two Inter Core 1.80GHz 1.99GHz CPUs and 16GB RAM.

Comparison Algorithms. To the best of our knowledge, there is no existing work on the MMCS problem. We compare and evaluate the following algorithms, i.e., MMCSearch$^\sharp$ (MMCSearch with only Lemma 3.1), MMCSearch* (MMCSearch with only Lemma 3.2), and MMCSearch (Algorithm 2 with all pruning rules). All the algorithms are implemented in Python.

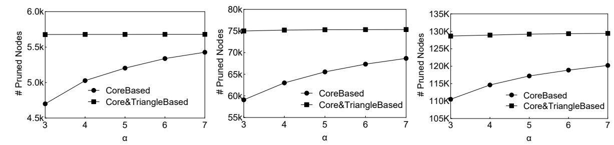
Datasets and Parameters. Three real-world signed networks are adopted in the experiments, i.e., Bitcoin(5.8K$*$35K), Slashdot(77K$*$516K) and Epinion (131K$*$841K), which are publicly available on SNAP (http://snap.stanford.edu). The number after each dataset represents the corresponding number of nodes and edges, i.e., $n * m$. The parameter k of our algorithms is settled from the interval [3, 7] with a default value of $k = 3$; α varies from the interval [3, 7] with a default value of $\alpha = 5$. Unless otherwise specified, when a parameter is varying, another parameter is set to its default value.

Exp1-Effectiveness Evaluation of SGReduction. In this experiment, we evaluate the effectiveness of the proposed SGReduction algorithm. Figure 3 reports the number of pruned nodes by CoreBased procedure, the sum of pruned nodes by CoreBased and TriangleBased when varying k and α. Clearly, TriangleBased prunes much more nodes than CoreBased, which proves that its pruning conditions are stricter. In Figure 3(a)(b)(c), as k increases, the number of pruned nodes by CoreBased increases as well. The same situation occurs when α increases in Figure 3(d)(e)(f). This is because as k or α increases, more nodes do not meet the requirement of the $(\alpha - 1, (k-1)\alpha)$-signed core in Lemma 3.1.

Exp 2-Efficiency Evaluation of MMCSearch. In this experiment, we evaluate the efficiency of three compared algorithms when varying k and α. As shown in Figure 4, the running time of all the algorithms decreases since as k or α increases, the power of all the proposed pruning rules strengthens. Hence, we can conclude that all the algorithms can complete the search in a short time on all datasets when varying k and α and MMCSearch significantly outperforms other algorithms which reveals the effectiveness of signed graph reduction and the efficiency of MMCSearch.

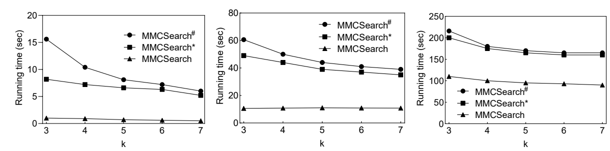


(a) Bitcoin (Vary k) (b) Slashdot (Vary k) (c) Epinion (Vary k)

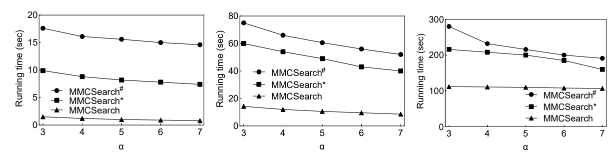


(d) Bitcoin (Vary α) (e) Slashdot (Vary α) (f) Epinion (Vary α)

Figure 3: Number of pruned nodes by SGReduction varying k or α



(a) Bitcoin (Vary k) (b) Slashdot (Vary k) (c) Epinion (Vary k)



(d) Bitcoin (Vary α) (e) Slashdot (Vary α) (f) Epinion (Vary α)

Figure 4: Running time of compared algorithms varying k or α

5 CONCLUSIONS

In this paper, we introduce a novel subgraph model, namely maximal multipolarized cliques, to characterize the multiple polarized structures in signed networks. To search all maximal multipolarized cliques, we first propose several powerful reduction techniques to substantially prune the signed network. Then, an efficient algorithm, MMCSearch, is developed to search all MMCs in the reduced signed network. The experimental results on real-world datasets demonstrate the efficiency and effectiveness of our algorithm.

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